Statistics

Section: Tests and Confidence Intervals

Exercises

1. Hypothesis and Null Hypothesis

1.1 How to derive the null-hypothesis

- It is __ to first look at the data and then frame H_0 to fit what the data show.

1	mathematically correct
2	an ordinary procedure
3	quite OK
4	natural
5	cheating

1.2 null-hypothesis and p-value

- Mark correct and false statements (Yes/No)

	A null hypothesis is a statement about a population.
	A null hypothesis is a statement about the sample.
	H_0: There is no difference in the true means.
	H_0: There is no difference in the sample means.
	H_0: $\mu = 0$
	H_0: $\bar{x} = 0$
	The p-value is the probability of getting an outcome as extreme as or more extreme than the actually observed outcome (sample) under the null hypothesis.
	Usually the null hypothesis is a statement of "no effect", "no difference" or "=0" and we are eager to find evidence against it.
	p-value = Prob(Sample | H_0)
	p-value = Prob(H_0 | Sample)
	Beware of searching for significance
	Legitimately you cannot test a hypothesis on those data that first suggested that hypothesis.
	When large samples are available, even tiny deviations from the null hypothesis will be significant.

1.3 Conclusions

There are two populations, e.g. two groups of farmers (traditional and organic farming technique)

H_0: Mean Income$_1$ = Mean Income$_2$

You draw a random sample of size 30 from each population.

Test-result: p = 0.96

Mark correct and false conclusions (Yes/No)

The mean income of the two farmer samples is equal.	
The mean income in the two farmer populations is equal.	
Traditional farmers have the same mean income as organic farmers.	
With probability 0.96, traditional farmers have the same mean income as organic farmers.	
With probability 0.04, traditional and organic farmers have the same mean income.	
The sample is not compatible (p=0.04) with the null hypothesis: the mean income in the two farmer populations is equal.	
We could not find a significant (at level 0.05) difference in mean income between the two farmer populations.	
Data did not show a significant (p=0.96) difference in mean income between traditional and organic farmers.	
Data are compatible (p=0.96) with the null hypothesis that traditional and organic farmers have the same mean income.	

Test-result: p = 0.04

Mark correct and false conclusions (Yes/No)

Mean income in the two farmer samples differs significantly (p=0.04).	
Mean income in the two farmer populations differs significantly (p=0.04).	
Traditional farmers have the same mean income as organic farmers (p=0.04).	
With probability 0.04, traditional and organic farmers have the same mean income.	
The null hypothesis, that the mean income of traditional and organic farmers are equal, is rejected at significance level α=0.05.	
The sample is not compatible (p=0.04) with the null hypothesis that the mean income of traditional and organic farmers is equal.	
Data are not compatible (p=0.04) with equal mean income gained by organic and traditional farming.	
Organic and traditional farmers have different mean income (p=0.04).	

2 Statistical significance and practical significance (relevance)

- A study comparing wages (each 5 male and female) reported
  mean difference (male-female) = -1.5$, p = 0.10, CI95% = [-2$; 1$]

- A study comparing wages (each 5 male and female) reported
  mean difference (male-female) = 260$, p = 0.10 CI95% = [-20$; 500$]

- A study comparing wages (each 500 male and female) reported
  mean difference (male-female) = -1.5$, p = 0.0001 CI95% = [-2$; -1$]

- A study comparing wages (each 500 male and female) reported
  mean difference (male-female) = 475$, p = 0.0001 CI95% = [450$; 500$]

- A randomized trial of interventions for reducing transmission of HIV-1 reported[1]:

  H_0 : incident rate[2]$_{intervention}$ / incident rate$_{control}$ = 1 (i.e. incident rate$_{intervention}$ = incident rate$_{control}$)

  The test was not significant at 0.05

  95% confidence interval was reported as [0.63 ; 1.58][3].

  Conclusion: As the test was not significant at 0.05 we conclude that intervention has no influence on the infection rate of HIV-1.

  What do you think?

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[1] Example from Moore (2006), p. 425
[2]Incident rate = rate of new infections
[3] Intervention might decrease the infection by 37% or increase the infection by 58%.

3 Confidence Interval

3.1 Definition

- Mark correct and false statements (Yes/No)

	A confidence interval always covers the true parameter.
	A confidence interval covers the true parameter with a given probability.
	A confidence interval covers the statistic with a given probability.
	In the long run, 95% of your confidence intervals will contain the true parameter

A confidence interval (CI) is determined by:
mean, standard deviation (Std), sample size (n) and the confidence level (C)

- Circle correct statements

All else being equal

n=100 CI is smaller / wider than   n=10 CI

Std=10 CI is smaller / wider than   Std = 100 CI

a 80% CI is smaller / wider than   a 95% CI

a 95% CI is smaller / wider than   a 99% CI

3.2 Sample size

- A pilot study reveals that the real value μ is around 100 and σ around 50

Determine the sample size so that a confidence interval derived from a sample of this size covers the real μ with C = 80%; 90%; 95%; 99.7% confidence level and has a width of less than 10% of the true parameter.

$$CI : \overline{x} \pm z \cdot SEM$$

C confidence level	rule of thumb z	exact z
80%	1.3	1.28
90%	1.6	1.64
95%	2	1.96
99%	2.6	2.56
99.7 %	3	2.97

$$\operatorname{Prob}(\overline{x} - z * SEM \leq \mu \leq \overline{x} + z * SEM) \geq C \Leftrightarrow \operatorname{Prob}(| \mu - \overline{x} | \leq z * \frac{s}{\sqrt{n}}) \geq C$$

Objective : $| \mu - \overline{x} | \leq 0.1 * \mu$

$$z * \frac{s}{\sqrt{n}} \leq 0.1 * \mu$$

$$n \geq \left(\frac{z * s}{0.1 * \mu} \right)^2$$